

Lösungen Semesterprüfung Mathematik 3. Klasse 2008/2009

Aufgabe 1

$$\begin{aligned} \frac{2x}{1-x} - \frac{2}{x-1} &= \frac{2x^2 - 6}{1-x^2} - \frac{8}{x+1} \Rightarrow \frac{2x}{1-x} + \frac{2}{1-x} = \frac{2x^2 - 6}{(1+x)(1-x)} - \frac{8}{1+x} \\ \Rightarrow D &= \mathbb{R} \setminus \{-1; 1\} \\ \Rightarrow 2x(1+x) + 2(1+x) &= 2x^2 - 6 - 8(1-x) \\ \Rightarrow 2x + 2x^2 + 2 + 2x &= 2x^2 - 6 - 8 + 8x \\ \Rightarrow 16 &= 4x \Rightarrow x = 4 \Rightarrow L = \{4\} \end{aligned}$$

Aufgabe 2

$$\begin{aligned} \frac{x+2}{x-5} \leq 3 &\Rightarrow \frac{x+2}{x-5} - 3 \leq 0 \Rightarrow \frac{x+2}{x-5} - \frac{3(x-5)}{x-5} \leq 0 \\ \Rightarrow \frac{x+2}{x-5} - \frac{3x-15}{x-5} &\leq 0 \Rightarrow \frac{x+2-3x+15}{x-5} \leq 0 \Rightarrow \frac{-2x+17}{x-5} \leq 0 \end{aligned}$$

	5		$\frac{17}{2}$
-2x+17	+	+	-
X-5	-	+	+

$$\Rightarrow L =]-\infty; 5[\cup \left[\frac{17}{2}; \infty \right[$$

Aufgabe 3

a) $m = \frac{\Delta y}{\Delta x} = \frac{4-0}{0-(-5)} = \frac{4}{5} \Rightarrow y = \frac{4}{5}x + q$

Punkt einsetzen, z.B. B:

$$\Rightarrow 4 = \frac{4}{5} \cdot 0 + q \Rightarrow q = 4 \Rightarrow y = \frac{4}{5}x + 4$$

b) $A' = A ; B'(0 / -4)$

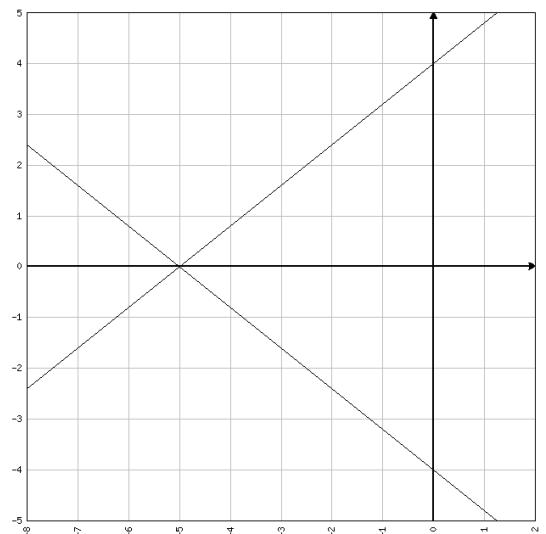
$$\Rightarrow m = \frac{\Delta y}{\Delta x} = \frac{-4-0}{0-(-5)} = -\frac{4}{5}$$

$$\Rightarrow y = -\frac{4}{5}x + q$$

Punkt einsetzen, z.B. B:

$$\Rightarrow -4 = \frac{4}{5} \cdot 0 + q \Rightarrow q = -4$$

$$\Rightarrow y = -\frac{4}{5}x - 4$$



$$c) g \cap h: y = \frac{4}{5} \cdot 3 + 4 = \frac{32}{5}$$

$$\Rightarrow P_1\left(3 / \frac{32}{5}\right)$$

$$g' \cap h: y = -\frac{4}{5} \cdot 3 - 4 = -\frac{32}{5}$$

$$\Rightarrow P_2\left(3 / -\frac{32}{5}\right)$$

Aufgabe 4

$$M = \frac{1}{4}x ; P = \frac{1}{5}x ; H = \frac{1}{6}x$$

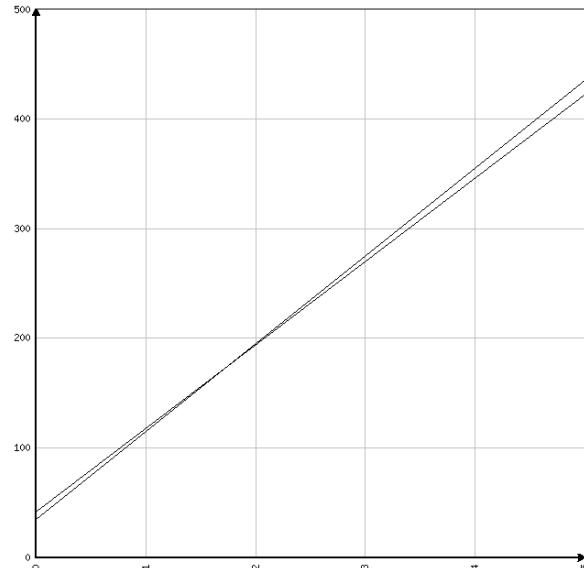
$$\frac{1}{4}x + \frac{1}{5}x + \frac{1}{6}x = 1850 \Rightarrow 15x + 12x + 10x = 111000 \Rightarrow x = 3000$$

$$\Rightarrow M = 750 ; P = 600 ; H = 500$$

Aufgabe 5

$$b) \text{ Firma A: } y = 76x + 42$$

$$\text{Firma B: } y = 80x + 35$$



$$a) \text{ Firma A: } y = 76 \cdot 3.5 + 42 = 308 ; \text{ Firma B: } y = 80 \cdot 3.5 + 35 = 315$$

$$b) 76x + 42 = 80x + 35 \Rightarrow x = \frac{7}{4} \Rightarrow \text{nach 1 h 45 min}$$

Aufgabe 6

g : Grundseite ; h : Höhe

$$\left| \begin{array}{l} \frac{gh}{2} + 65 = \frac{(g+5)(h+2)}{2} \\ \frac{gh}{2} - 7 = \frac{(g+3)(h-2)}{2} \end{array} \right| \Rightarrow \left| \begin{array}{l} gh + 130 = gh + 2g + 5h + 10 \\ gh - 14 = gh - 2g + 3h - 6 \end{array} \right|$$

$$\text{II: } 2g = 120 - 5h \Rightarrow \text{I: } -14 = -(120 - 5h) + 3h - 6$$

$$\Rightarrow 112 = 8h \Rightarrow h = 14 \Rightarrow g = 25$$

Aufgabe 7

$$\text{Ordnen } \Rightarrow \begin{vmatrix} x - 2y - 2z = 1 \\ -2x + y - 6z = 2 \\ x - 3y + z = -4 \end{vmatrix}$$

$$L = \{(3/2/-1)\}$$

Aufgabe 8

$$\begin{vmatrix} ax + bx + ay - by = b \\ ax - bx - ay - by = b \end{vmatrix}$$

Durch Subtraktion erhält man eine einfache 3. Gleichung $\Rightarrow I - II:$

$$2bx + 2ay = 0 \Rightarrow x = -\frac{ay}{b} \Rightarrow \text{In I:}$$

$$-a \cdot \frac{ay}{b} - b \cdot \frac{ay}{b} + ay - by = b \Rightarrow -a^2y - b^2y = b^2$$

$$\Rightarrow x = -\frac{b^2}{a^2 + b^2} \Rightarrow L = \left\{ \left(\frac{ab}{a^2 + b^2}, -\frac{b^2}{a^2 + b^2} \right) \right\}$$

Aufgabe 9

$$a) \frac{x}{a+x} = \frac{d}{b} \Rightarrow bx = ad + dx \Rightarrow bx - dx = ad \Rightarrow x(b-d) = ad$$

$$\Rightarrow x = \frac{ad}{b-d}$$

$$b) x = \frac{20 \cdot 24}{34 - 24} = 48 \text{ m}$$

Aufgabe 10

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