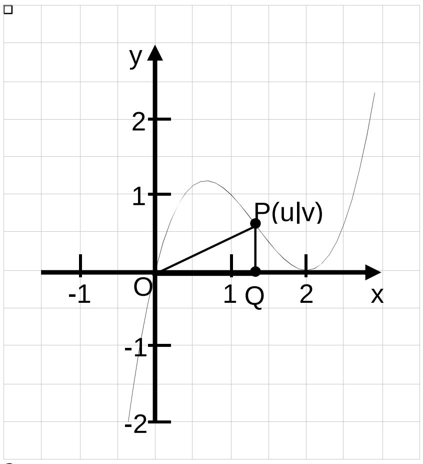
1 Extremwertaufgabe, Integral (8 P)



a) Nullstellen: $x^3 - 4x^2 + 4x = 0$ **(2 P)**

$\Leftrightarrow x(x^2 - 4x + 4) = 0 \;\Leftrightarrow x(x-2)^2 = 0$
$x_1 = 0,\; x_2 = 2$

b) $A = \int_0^2 (x^3 - 4x^2 + 4x)dx = \left[\frac{1}{4}x^4 - \frac{4}{3}x^3 + 2x^2\right]_0^2 = 4 - \frac{32}{3} + 8 - 0 = \frac{4}{3}$ **(2 P)**

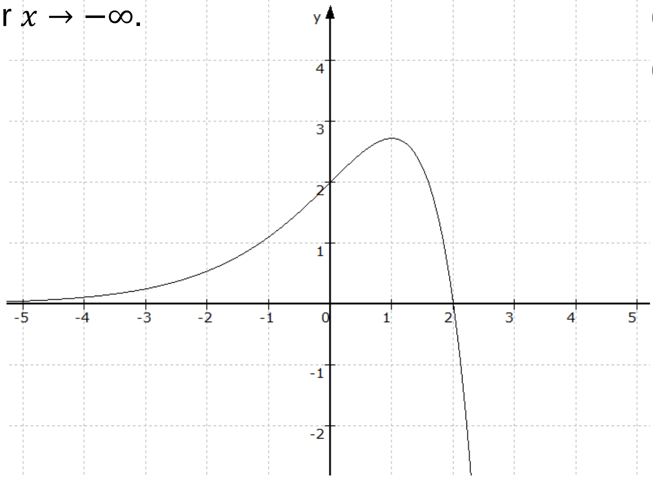
c) Zielfunktion: $F(u,v) = \frac{1}{2}uv$
NB: $P(u|v) \in G_f \Leftrightarrow v = u^3 - 4u^2 + 4u$
$\Rightarrow F(u) = \frac{1}{2}u(u^3 - 4u^2 + 4u) = \frac{1}{2}u^4 - 2u^3 + 2u^2$, wobei $ID_u =]0;2[$
$F'(u) = 2u^3 - 6u^2 + 4u = 0 \Rightarrow 2u(u^2 - 3u + 2) = 0 \Rightarrow 2u(u-1)(u-2) = 0$
$\Rightarrow u_1 = 0,\; u_2 = 1,\; u_3 = 2$
$F''(u) = 6u^2 - 12u + 4 \Rightarrow F''(1) = 6 - 12 + 4 = -2 < 0 \Rightarrow$ bei $u_2 = 1$ lok. Max.
Bei den offenen Randstellen $u_1 = 0$ und $u_3 = 2$ von $ID_u =]0;2[$ gilt $\lim_{x\to 0^+} F(x) = 0$ und $\lim_{x\to 2^-} F(x) = 0$ (oder aus Skizze)
$F(1) = \frac{1}{2} - 2 + 2 = \frac{1}{2} \Rightarrow$ bei $u_2 = 1$ absolutes Max. **(4 P)**

2 Differential-, Integralrechnung (13 P)

a) $f'(x) = \left((2-x)\cdot e^x\right)' = -e^x + (2-x)\cdot e^x = (-1+2-x)\cdot e^x = (1-x)\cdot e^x$
$f''(x) = \left((1-x)\cdot e^x\right)' = -e^x + (1-x)\cdot e^x = (-1+1-x)\cdot e^x = -x\cdot e^x$ gilt. **(2 P)**

b) $f'''(x) = (-x\cdot e^x)' = -e^x - x\cdot e^x = (-1-x)\cdot e^x = -(x+1)\cdot e^x$
Nullstellen: $f(x) = 0 \Rightarrow (2-x)\cdot e^x = 0 \Rightarrow x = 2$
Extremalpunkte: $f'(x) = 0 \Rightarrow (1-x)\cdot e^x = 0 \Rightarrow x = 1,\; f''(1) = -e < 0 \Rightarrow H(1|e)$
Wendepunkte: $f''(x) = 0 \Rightarrow -x\cdot e^x = 0 \Rightarrow x = 0,\; f'''(0) = -1 \neq 0 \Rightarrow W(0|2)$
Asymptoten: $\lim_{x\to\infty}\left((2-x)\cdot e^x\right) = -\infty,\; \lim_{x\to-\infty}\left((2-x)\cdot e^x\right) = 0$
$\Rightarrow y = 0$ Asymptote für $x \to -\infty$. **(4 P)**

c) **(1 P)**



d) $F = \int_0^2 f(x)\,dx = \int_0^2 (2-x)\cdot e^x\,dx$, part. Integration: $u(x) = 2-x$, $v'^{(x)} = e^x$, $v(x) = e^x$

$\int (2-x)\cdot e^x\,dx = (2-x)e^x - \int(-1)\cdot e^x\,dx = (2-x)e^x + \int e^x\,dx$

$= (2-x)e^x + e^x + C = (3-x)e^x + C, C \in \mathbb{R}$

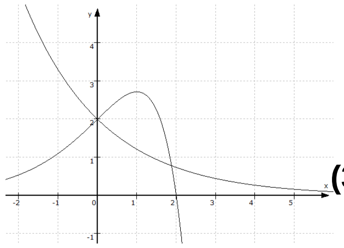
$\Rightarrow F = \int_0^2 (2-x)\cdot e^x\,dx = [(3-x)e^x]_0^2 = e^2 - 3 \cong 4.389$ **(3 P)**

e) $(0|2) \in G_g \Rightarrow g(0) = a\cdot e^0 = a = 2 \Rightarrow g(x) = 2e^{bx}$

$g'(x) = 2b\cdot e^{bx} \Rightarrow m_g = g'(0) = 2b$

$f'(x) = (1-x)\cdot e^x \Rightarrow m_f = f'(0) = 1$

$m_f \cdot m_g = -1 \Rightarrow 1\cdot 2b = -1 \Rightarrow b = -\frac{1}{2} \Rightarrow g(x) = 2e^{-\frac{1}{2}x}$ **(3 P)**



3 Wahrscheinlichkeit (6 P)

a) $P(rgrgr) = \left(\frac{3}{4}\right)^3 \cdot \left(\frac{1}{4}\right)^2 = \frac{27}{1024} \cong 0.026$ **(1 P)**

b) X: Anzahl „grün"

$P(X=3) = \binom{5}{3}\left(\frac{1}{4}\right)^3 \cdot \left(\frac{3}{4}\right)^2 = \frac{90}{1024} \cong 0.0879$ **(1 P)**

c) $P(A=5) = P(X=3) = \frac{90}{1024}$, $P(A=15) = P(rgrgr) = \frac{27}{1024}$,

$P(A=30) = P(X=4) = \binom{5}{4}\left(\frac{1}{4}\right)^4 \cdot \left(\frac{3}{4}\right)^1 = \frac{15}{1024}$, $P(A=100) = P(X=5) = \left(\frac{1}{4}\right)^5 = \frac{1}{1024}$,

$P(A=0) = \frac{1024-90-27-15-1}{1024} = \frac{891}{1024}$

a_i	5	15	30	100	0
$P(A=a_i)$	$\frac{90}{1024}$	$\frac{27}{1024}$	$\frac{15}{1024}$	$\frac{1}{1024}$	$\frac{891}{1024}$

(3 P)

d) $E(A) = \frac{450+405+450+100+0}{1024} = \frac{1405}{1024} \cong 1.372 \Rightarrow E(Einsatz - A) \cong 2 - 1.372 = 0.628$

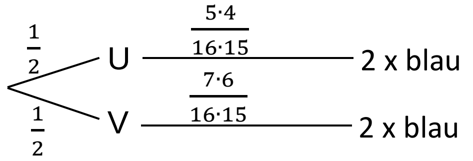
Der Betreiber kann einen Erlös von rund 63 Rappen erwarten (pro Spiel). **(1 P)**

4 Wahrscheinlichkeit (4 P)

a) $P(bb\ aus\ Urne\ V) = \frac{7\cdot 6}{16\cdot 15} = \frac{42}{240} = \frac{7}{40} = 0.175$ **(1 P)**

b) $P(gleichfarbig\ aus\ Urne\ U) = \frac{11\cdot 10}{16\cdot 15} + \frac{5\cdot 4}{16\cdot 15} = \frac{130}{240} = \frac{13}{24} \cong 0.5417$ **(1 P)**

c)



$p = \frac{1}{2}\cdot\frac{1}{12} = \frac{1}{24} = \frac{10}{240}$

$p = \frac{1}{2}\cdot\frac{7}{40} = \frac{7}{80} = \frac{21}{240}$

$P(2\ x\ blau) = \frac{10}{240} + \frac{21}{240} = \frac{31}{240}$

$P_{2\ x\ blau}(aus\ Urne\ U) = \frac{P(2\ x\ blau \wedge aus\ U)}{P(2\ x\ blau)} = \frac{\frac{10}{240}}{\frac{31}{240}} = \frac{10}{31} \cong 0.3226$ **(2 P)**

5 Vektorgeometrie (12 P)

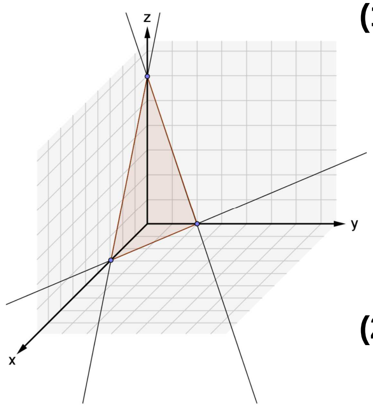
a) $\overrightarrow{CD} = \begin{pmatrix} -2 \\ -3 \\ -6 \end{pmatrix} \Rightarrow \overline{CD} = \sqrt{(-2)^2 + (-3)^2 + (-6)^2} = \sqrt{49} = 7$

$\overrightarrow{OC} = \begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix}, \overrightarrow{OD} = \begin{pmatrix} 1 \\ -2 \\ -2 \end{pmatrix} \Rightarrow cos\varphi = \frac{3\cdot1+1\cdot(-2)+4\cdot(-2)}{\sqrt{3^2+1^2+4^2}\cdot\sqrt{1^2+(-2)^2+(-2)^2}} = -\frac{7}{3\sqrt{26}}$

$\Rightarrow \varphi = \arccos\left(-\frac{7}{3\sqrt{26}}\right) \cong 117.2°$ **(2 P)**

b) $2 \cdot 1 + 3 \cdot (-2) + z \cdot (-2) - 6 = -6 - 6 \neq 0 \Rightarrow D \notin E$ **(1 P)**

c) $u: y = z = 0,\ x = u \implies 2u - 6 = 0 \implies u = 3$
$v: x = z = 0,\ y = v \implies 3v - 6 = 0 \implies v = 2$
$w: x = y = 0,\ z = w \implies w - 6 = 0 \implies w = 6$



(2 P)

d) $\vec{n} = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$, Normale zu E durch C: $n: \vec{r} = \begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix} + t \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$

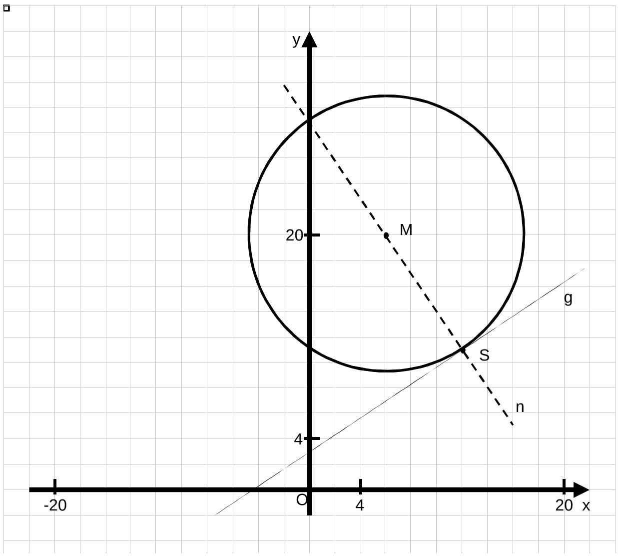
t für Durchstosspunkt S von n mit E:
$2x + 3y + z - 6 = 0$
$2(3 + 2t) + 3(1 + 3t) + 4 + t - 6 = 0 \Rightarrow 14t + 7 = 0 \Rightarrow t = -\frac{1}{2}$

$\Rightarrow \overrightarrow{r_{C'}} = \begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix} + 2 \cdot \left(-\frac{1}{2}\right) \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} \Rightarrow C'(1|-2|3)$ **(3 P)**

e) $A(x|0|0),\ B(0|y|0) \Rightarrow \overrightarrow{AB} = \begin{pmatrix} -x \\ y \\ 0 \end{pmatrix}, \overrightarrow{CA} = \begin{pmatrix} x-3 \\ -1 \\ -4 \end{pmatrix}, \overrightarrow{CB} = \begin{pmatrix} -3 \\ y-1 \\ -4 \end{pmatrix}$

$\gamma = \measuredangle ACB = 90° \Rightarrow \overrightarrow{CA} \cdot \overrightarrow{CB} = 0 \Rightarrow -3x + 9 - y + 1 + 16 = 0 \Rightarrow y = 26 - 3x$
$c = \overline{AB} = 10 \Rightarrow (-x)^2 + y^2 = x^2 + y^2 = 100$
$\Rightarrow x^2 + (26 - 3x)^2 = 100 \Rightarrow 10x^2 - 156x + 576 = 0 \Rightarrow 5x^2 - 78x + 288 = 0$
$\Rightarrow x_{1/2} = \frac{78 \pm \sqrt{78^2 - 20\cdot288}}{10} = \frac{78 \pm 18}{10}$
$\Rightarrow x_1 = \frac{96}{10} = 9.6,\ y_1 = 26 - 3 \cdot 9.6 = -2.8 \Rightarrow A_1(9.6|0|0),\ B_1(0|-2.8|0)$
$x_2 = \frac{60}{10} = 6,\ y_2 = 26 - 3 \cdot 6 = 8 \Rightarrow A_2(6|0|0),\ B_2(0|8|0)$ **(4 P)**

6 Kreis (4 P)



Normale n zu g durch M: $y = -\frac{3}{2}x + b$

$\overset{M(6|20)}{\Longrightarrow} 20 = -9 + b \Rightarrow b = 29$

$\Rightarrow y = -\frac{3}{2}x + 29$

$g \cap n$: $\frac{2}{3}x + 3 = -\frac{3}{2}x + 29 \Rightarrow (\frac{2}{3} + \frac{3}{2})x = 26$

$\Rightarrow \frac{13}{6}x = 26 \Rightarrow x = 12 \Rightarrow y = \frac{2}{3} \cdot 12 + 3 = 11$

Schnittpunkt $S(12|11)$

Radius von k:

$r = \overline{MS} = \sqrt{(12-6)^2 + (11-20)^2} =$

$\sqrt{36 + 81} = \sqrt{117}$

Gleichung von k: $(x-6)^2 + (y-20)^2 = 117$

(4 P)

7 Folgen und Reihen (3 P)

Folge der Durchmesser: $d_n = 80 \cdot \left(\frac{3}{5}\right)^{n-1}$

Folge der Abstände der Endpunkte der Halbkreise zu A:

$s_n = 80 - 80 \cdot \frac{3}{5} + 80 \cdot \left(\frac{3}{5}\right)^2 - \cdots \pm 80 \cdot \left(\frac{3}{5}\right)^{n-1} = \sum_{i=1}^{n} 80 \cdot \left(-\frac{3}{5}\right)^{i-1}, \left|-\frac{3}{5}\right| < 1$

Der Grenzwert s von s_n ist die Entfernung von E zu A:

$s = \lim_{n\to\infty} s_n = \sum_{i=1}^{\infty} 80 \cdot \left(-\frac{3}{5}\right)^{i-1} = 80 \cdot \frac{1}{1-\left(-\frac{3}{5}\right)} = 80 \cdot \frac{5}{8} = 50$ **(3 P)**