## 1 Differenzialrechnung I

a) $f'(x) = 0 \Rightarrow -x^2 + 3 = 0 \Rightarrow x_{4,5} = \pm\sqrt{3}$

$f''(x) = -2x\ ,\ f''(\sqrt{3}) < 0 \Rightarrow\ H$

$f(\sqrt{3}) = -\sqrt{3} + 3\sqrt{3} = 2\sqrt{3} \Rightarrow H(\sqrt{3}|2\sqrt{3})$

Symmetrie bezüglich $(0|0) \Rightarrow T(-\sqrt{3}|-2\sqrt{3})$

b) $f'(x) = p'(x) \Leftrightarrow -x^2 + 3 = 2x \Leftrightarrow (x+3)(x-1) = 0 \Rightarrow x_7 = -3\ ,\ x_8 = 1$

$f(x) = p(x) \Leftrightarrow\ -\frac{1}{3}x^3 + 3x = x^2 + k \Leftrightarrow k = -\frac{1}{3}x^3 - x^2 + 3x$

$x = -3 \Rightarrow k_1 = -9\ ,\ x = 1 \Rightarrow k_2 = \frac{5}{3}$

c) 1. Hauptbedingung: $u = 2a + 2b$

2. Nebenbedingung: $b = -\frac{1}{3}a^3 + 3a$

3. Zielfunktion: $u(a) = 2a + 2\left(-\frac{1}{3}a^3 + 3a\right) = -\frac{2}{3}a^3 + 8a\ ,\ D = [0;3]$

4. Extremum: $u'(a) = -2a^2 + 8 = 0 \Rightarrow a_1 = -2 \notin D\ ,\ a_2 = 2 \in D$

5. Lösung: $u''(a) = -4a < 0$ für $a > 0 \Rightarrow$ Maximum für $a = 2 \Rightarrow P\left(2\middle|\frac{10}{3}\right)$

$u_{max} = u(2) = -\frac{16}{3} + 16 = \frac{32}{3} = 10.\overline{6}$

## 2 Differenzialrechnung II

a) $D = \mathbb{R}_0^+$

b) $\lim\limits_{x\to\infty}\left(\sqrt{x}\cdot e^{-\frac{1}{2}x}\right) = 0$ ($e^{-\frac{1}{2}x}$ dominiert $\sqrt{x}$)

c) $f'(x) = \frac{1}{2\sqrt{x}}\cdot e^{-\frac{1}{2}x} + \sqrt{x}\cdot e^{-\frac{1}{2}x}\cdot\left(-\frac{1}{2}\right) = e^{-\frac{1}{2}x}\left(\frac{1}{2\sqrt{x}} - \frac{\sqrt{x}}{2}\right)$

d) $m_t = f'(4) = e^{-2}\left(\frac{1}{4} - 1\right) = -\frac{3}{4e^2} \Rightarrow t{:}\ y = -\frac{3}{4e^2}x + b$

$P\left(4|\frac{2}{e^2}\right) \in t \Rightarrow\ \frac{2}{e^2} = -\frac{3}{e^2} + b \Rightarrow b = \frac{5}{e^2} \Rightarrow t{:}\ y = -\frac{3}{4e^2}x + \frac{5}{e^2}$

## 3 Integralrechnung I

Schnittstellen: $x^2 - ax = ax \Leftrightarrow x^2 - 2ax = 0, x(x-2a) = 0 \Leftrightarrow x = 0$ oder $x = 2a$

$\Rightarrow A(a) = \int_0^{2a}\left(g_a(x) - f_a(x)\right)dx = \int_0^{2a}(2ax - x^2)dx = \left[ax^2 - \frac{x^3}{3}\right]_0^{2a} = 4a^3 - \frac{8a^3}{3} = \frac{4a^3}{3}$

Es muss gelten: $\frac{4a^3}{3} = 36 \Leftrightarrow a^3 = 27 \Leftrightarrow a = 3$

## 4 Integralrechnung II

a) $g$: Nullstelle $x_1 = -1$ $f$: Nullstelle $x_2 = 9$

b) $\sqrt{9-x} = 3\sqrt{x+1} \quad |(\,)^2 \Rightarrow 9 - x = 9(x+1) \Rightarrow x = 0 \Rightarrow S(0|3)$

c) $V = \pi \int_{-1}^{0} 9(x+1)dx + \pi \int_{0}^{9}(9-x)dx = \pi \left[\frac{9}{2}x^2 + 9x\right]_{-1}^{0} + \pi \left[9x - \frac{1}{2}x^2\right]_{0}^{9} = 45\pi$

## 5 Vektorgeometrie

a) $\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{AD} = \overrightarrow{OA} + \overrightarrow{BC} = \begin{pmatrix} 7 \\ -7 \\ 4 \end{pmatrix} + \begin{pmatrix} -12+1 \\ 10-3 \\ 1-0 \end{pmatrix} = \begin{pmatrix} -4 \\ 0 \\ 5 \end{pmatrix} \Leftrightarrow D(-4|0|5)$

b) $\alpha = \arccos\left(\frac{\overrightarrow{BA}\cdot\overrightarrow{BC}}{|\overrightarrow{BA}|\cdot|\overrightarrow{BC}|}\right) = \arccos\left(\frac{\begin{pmatrix} 8 \\ -10 \\ 4 \end{pmatrix}\cdot\begin{pmatrix} -11 \\ 7 \\ 1 \end{pmatrix}}{\sqrt{64+100+16}\cdot\sqrt{121+49+1}}\right) = \arccos\left(\frac{-88-70+4}{\sqrt{180}\cdot\sqrt{171}}\right) \cong 151.4\,°$

c) $Q(x|0|0)\,,\ |\overrightarrow{QA}| = |\overrightarrow{QB}| \Rightarrow \left|\begin{pmatrix} 7-x \\ -7 \\ 4 \end{pmatrix}\right| = \left|\begin{pmatrix} -1-x \\ 3 \\ 0 \end{pmatrix}\right|$

$\Rightarrow \sqrt{(7-x)^2 + (-7)^2 + 4^2} = \sqrt{(-1-x)^2 + 3^2}$

$\Rightarrow 49 - 14x + x^2 + 49 + 16 = 1 + 2x + x^2 + 9$

$\Leftrightarrow x^2 - 14x + 114 = x^2 + 2x + 10 \Leftrightarrow 104 = 16x \Leftrightarrow x = 6.5 \Rightarrow Q(6.5|0|0)$

d) Senkrechte $g$ zu $E$ durch $P \Rightarrow g\colon \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1.5 \\ -6 \\ 1.5 \end{pmatrix} + t\begin{pmatrix} 7 \\ -2 \\ -3 \end{pmatrix}$

$g \cap E \Rightarrow 7(1.5+7t) - 2(-6-2t) - 3(1.5-3t) + 13 = 0 \Rightarrow t = -\frac{1}{2}$

$\Rightarrow \overrightarrow{OP'} = \begin{pmatrix} 1.5 \\ -6 \\ 1.5 \end{pmatrix} + 2\cdot\left(-\frac{1}{2}\right)\cdot\begin{pmatrix} 7 \\ -2 \\ -3 \end{pmatrix} = \begin{pmatrix} -5.5 \\ -4 \\ 4.5 \end{pmatrix} \Leftrightarrow P'(-5.5|-4|4.5)$

## 6 Wahrscheinlichkeit I

a) $P(\text{5 blaue Kugeln}) = \frac{8}{10}\cdot\frac{7}{9}\cdot\frac{6}{8}\cdot\frac{5}{7}\cdot\frac{4}{6} = \frac{2}{9}$

b) $P(\text{2 rote Kugeln} \mid \text{2 gleichfarbige Kugeln}) = \frac{\frac{2}{10}\cdot\frac{1}{9}}{\frac{2}{10}\cdot\frac{1}{9}+\frac{8}{10}\cdot\frac{7}{9}} = \frac{1}{29}$

c) $P(\text{2 rote Kugeln}) = \binom{5}{2}\cdot\left(\frac{1}{5}\right)^2\cdot\left(\frac{4}{5}\right)^3 = \frac{128}{625} = 0.2048$

d) $P(\text{mind. 1 rote Kugel}) > 0.9 \Leftrightarrow 1 - P(\text{keine rote Kugel}) > 0.9 \Leftrightarrow 0.1 > 0.8^n$

$\Leftrightarrow -1 > n\cdot\lg(0.8) \Leftrightarrow -\frac{1}{\lg(0.8)} < n \Leftrightarrow 10.3\ldots < n \Rightarrow n = 11$

## 7 Wahrscheinlichkeit II

a) Wahrscheinlichkeitsverteilung für den Gewinn $X$:

| $x_i$ | $-5$ | $1$ | $2$ | $3$ |
|---|---|---|---|---|
| $p_i$ | $\left(\frac{2}{3}\right)^3 = \frac{8}{27}$ | $\frac{1}{3} = \frac{9}{27}$ | $\frac{2}{3} \cdot \frac{1}{3} = \frac{6}{27}$ | $\left(\frac{2}{3}\right)^2 \cdot \frac{1}{3} = \frac{4}{27}$ |

$E(X) = \frac{8}{27} \cdot (-5) + \frac{9}{27} \cdot 1 + \frac{6}{27} \cdot 2 + \frac{4}{27} \cdot 3 = -\frac{7}{27} \cong -0.26$ Franken

b) $P(X = 2) = (1 - p) \cdot p = 0.24 \Leftrightarrow p^2 - p + 0.24 = 0 \Leftrightarrow (p - 0.4)(p - 0.6) = 0$

$\Rightarrow p_1 = 0.4 \Leftrightarrow \alpha_1 = 0.4 \cdot 360° = 144°$ , $p_2 = 0.6 \Leftrightarrow \alpha_2 = 0.6 \cdot 360° = 216°$

## 8 Folgen und Reihen

Summe: $x + y + z = 38$ (I)

AF: $z - 2 - y = y - x \Leftrightarrow x - 2y + z = 2$ (II)

GF: $\frac{z}{y} = \frac{y}{x} \Leftrightarrow xz = y^2$ (III)

I − II $\Rightarrow 3y = 36 \Leftrightarrow y = 12$

In I und III $\Rightarrow z = 26 - x$ (IV), $xz = 144$ (V)

IV in V $\Rightarrow x(26 - x) = 144 \Leftrightarrow x^2 - 26x + 144 = 0 \Leftrightarrow (x - 8)(x - 18) = 0$

$x_1 = 8 \Rightarrow 8, 12, 18$ , $x_2 = 18 \Rightarrow 18, 12, 8$